KSME Journal, Vol. 1, No. 1, pp.31~35, 1987.

DYNAMIC INSTABILITY ANALYSIS OF A NON-AUTONOMOUS SYSTEM SUBJECTED TO MOVING IMPULSIVE NON-CONSERVATIVE LOAD

Min-Jae Shin*, Hun Heo* and Joo Hoh*
(Received January 26, 1987)

When a force is applied tangentially on the main axis of beam-type structure, it is well known that the system becomes non-conservative.

Dynamic instability of tube excited by moving follower force discretely and periodically is investigated. The effective stiffness of system becomes non-stationary due to time-dependent follower force.

Floquet theory is employed to examine the heteroparametric instability regions in the system parameter space. It turns out that there exist two type of instability that are due to parametric and combination resonances, which are confirmed through numerical simulation. Unique features which occur in parametric vibration are also discussed.

Key Words : Moving Follower Force, Non-Conservative System, Non-Autonomous System, Floquet Theory, Dynamic Instability, Combination Resonance, Parametric Resonance

1. INTRODUCTION

When a force is acting on the main axis of structure tangentially, then the force is referred to as a follower force. It has been a great concern for dynamicists to analyse the peculiar phenomena occuring in the system under the influence of follower force for many years(Bolotin, 1963, Leipholz, 1970, Leipholz, 1980, Ziegler, 1968). A great deal of effort has been made to investigate intensively on the dynamics and stability of the beam-type structure subjected to follower force, which makes the system be non-conservative(Bolotin, 1964, Leipholz, 1975). Many of the investigations were concerned with beam-like structure excited on its tip by continuous compression type non-conservative load(Bolotin 1963, Ro 1986, Tso, 1971, Ziegler, 1968). From the early 70's, researchers began to consider the system, such as pipe, excited by the continuous tension type non-conservative load which is due to the pulsating outflow effect of fluid(Chen, 1971, Ginsberg, 1973). However, to the best of authors' knowledge, dynamic characteristics of the non-conservative system due to moving tension-type follower force applied in discrete manner have not been investigated yet(Ibrahim, 1978a, Ibrahim, 1978b). In this paper, heteroparameric instability of a continuously flexible cantilevered pipe due to moving tension type non-conservative load acting discretely and periodically is investigated.

2. THE SYSTEM AND GOVERNING EQUATION

Consider a uniform flexible pipe of length ℓ, cross-sectional area A, density per unit length ρ, and flexural rigidity EI (Fig. 1).

Two-dimensional cartesian coordinate is employed to represent the behavior of the system. Periodically moving follower force is applied to the system in discrete manner (Fig. 2). It is well known that for any arbitrary virtual displacement of a system, the combined virtual work of real forces and inertia forces must vanish. That is,

$$\delta W \equiv \delta W_{n.c} - \delta V + \delta W_{\text{inertia force}} \tag{1}$$

where

$W_{n.c}$: Work done by non-conservative force

V : Potential energy

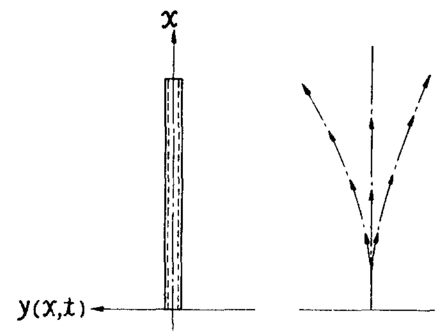

Fig. 1 Schematic representation of the system model

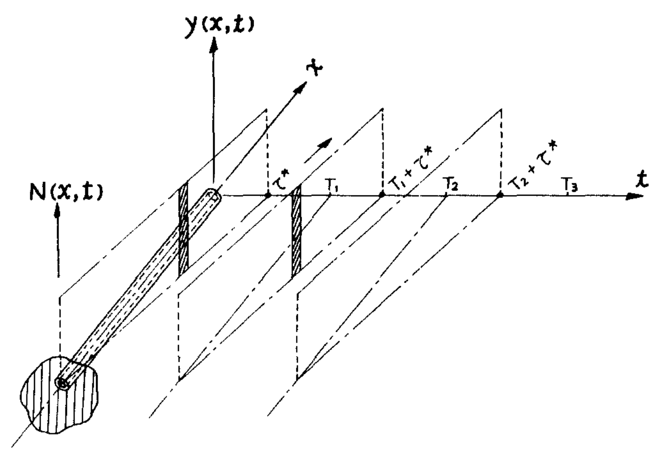

Fig. 2 Schematic representation of moving impulsive non-conservative load

For the beam of single degree of freedom model

$$\delta W_{\text{inertia force}} = -\int_0^{\ell} \rho A \ddot{y} dx \delta y \tag{2}$$

$$\delta V = \left[\int_0^{\ell} EI(x)(y'')^2 dx\right] y \delta y$$

$$\delta W_{n.c} = -\int_0^{\bar{x}} N(x,t)(y')^2 dx \delta y$$

$$= -N_0(t)\int_0^{\bar{x}} (y')^2 dx \delta y$$

where

N(x, t) ; Moving follower force, which is constant $N_0(t)$ throughout the whole length of the system

$\bar{x} = x(t)$; $0 \leq \bar{x} \leq \ell$

′ ; Derivative with respect to x

* Dae Jeon Machine Depot, Daejeon 300, Korea

$\cdot$; Derivative with respect to t

the following can be obtained by substituting Eq. (2) into Eq. (1).

$$m\ddot{y}+(K+K_G)y=0 \qquad (3)$$

where

$$m=\int_0^l \rho A(\bar{y})^2 dx$$

$$K=\int_0^l EI(y'')^2 dx \qquad (4)$$

$$K_G=N_o(t)\int_0^{\bar{x}}(y')^2 dx$$

$=k_G(t)$; Time dependent geometric stiffness due to moving non-conservative load.

As is seen in Eq. (3), the effect of non-conservative load appears as a geometric stiffness, which makes effective stiffness of the system be nonstationary. Expanding Eq. (3) in 3 mode approximation as follows, adopting the following non-dimensional parameters,

$$y(x,t)=\sum_{i=1}^{3}\phi_i(x)\, q_i(t) \qquad (5)$$

where

$$\phi_i(x)=\sin\frac{2i-1}{2}\pi\frac{x}{\ell}$$

$$\eta=\frac{x}{\ell},\quad \tau=\frac{t}{T^*} \qquad (6)$$

where

$$T^*=\sqrt{\rho A\ell^4/EI}$$

with the orthogonality relationship of normal modes, equation of motion in normal coordinate becomes

$$[M]\{\ddot{q}\}+[K+K_G]\{q\}=\{0\} \qquad (7)$$

where

$$M_{ij}=\delta_{ij}\int_0^1 \phi_i\phi_j d\eta$$

$$K_{ij}=\delta_{ij}\int_0^1 \phi_i''\phi_j'' d\eta$$

$$K_{Gij}=\delta_{ij}\, N_o(\tau)\int_0^{\bar{\eta}}\phi_i'\phi_j' d\eta$$

$$=K_{Gii}(\tau)$$

$0\le\bar{\eta}\le 1,\ i,j=1,2,3$

$$\delta_{ij}=\begin{cases}1 & i=j\\ 0 & i\neq j\end{cases}$$

$N(\eta,\tau)=N_o$: $0<\tau<\tau^*$

$=0$: $\tau^*<\tau<T_P$

$\bar{\eta}(\tau)=\int_0^{\tau} e^{w\xi}d\xi$; $0<\tau<\tau^*$

N_o : Constant follower force

w : Velocity coefficient of the moving follower force

τ^* : The time duration of moving follower force in one period

$\hat{}$: Derivative with respect to non-dimensional time

$T_P=T/T^*$: Non-dimensional period of non-conservative loading

T : Period of non-conservative loading

Equation (7) is a well-known Mathieu-Hill type equation in decoupled form for multi degree of freedom system.

In a compact form,

$$\{\ddot{q}\}+a_i[1+\alpha_i b_{ii}(\tau)]\{q\}=\{0\} \qquad (8)$$

where

a_i : Eigenvalue of the i-th mode

$$\alpha_i=\alpha_o\cdot g_i=\frac{4\ell^2 N_o}{EI\pi^2}\Big/\int_0^1 \phi_i'^2(\eta)d\eta$$

: system parameter for non-conservative load in the i-th mode.

$b_{ii}(\tau)=b_{ii}(\tau+T_P)$: Periodically varying non-conservative load

Introducing normal mode damping coefficient C_i and by using state variables $\{z\}$, equation of motion is obtained in an abbreviated form as follows,

$$\{\hat{z}\}=\begin{bmatrix}[A^{11}] & [A^{12}]\\ [A^{21}] & [A^{22}]\end{bmatrix}\{z\}=[A(\tau)]\{z\} \qquad (9)$$

where

$$\{Z\}=\begin{Bmatrix}q\\ \hat{q}\end{Bmatrix}$$

$[A^{11}]=[0]$

$[A^{12}]=[I]$

$[A^{21}]=\delta_{ij}[-a_i\langle 1+\alpha_i b_{ii}(\tau)\rangle]$

$[A^{22}]=\delta_{ij}[-C_i]$

$i,j=1,2,3$

$[A(\tau)]=[A(\tau+T_P)]$: Periodic matrix(6×6)

3. THEORY AND APPLICATION

When certain properties of a physical system vary with time, the governing equation of the physical system contains time-dependent coefficients. The system is then said to be non-autonomous. Since the mathematical treatment of non-autonomous system is considerably complicated, as compared to an autonomous system, the solution technique is concerned primarily with the form of solution rather than the exact solution.

Among the many forms of non-autonomous systems, the linear homogeneous system with periodic coefficients is of particular significance. The solution technique of such a system described below is frequently referred to as Floquet theory(Cesari, 1971, Coddington, 1955, McLachlan, 1964).

Following linear homogeneous system equation with time dependent coefficient is considered

$$\{\hat{Z}\}=[A(\tau)]\{Z\} \qquad (10)$$

where

$A(\tau)$ is periodic functions of time with period T :

i.e., $[A(\tau+T)]=A(\tau)]$

Following is the theorem due to Floquet.

If $[\phi(\tau)]$ is a fundamental matrix of the system described by (10), then $[\phi(\tau+T)]$ is also a fundamental matrix. Moreover, for every such $[\phi(\tau)]$ there exist a periodic nonsingular matrix $[A(\tau)]$ with period T and a constant matrix $[R]$ such that

$$[\phi(\tau)]=[A(\tau)]\exp(\tau[R]) \qquad (11)$$

where

$[A(\tau)]=[A(\tau+T)]$

Considering the theorem above, following can be formed

$$[\phi(\tau+T)]=[\phi(\tau)][C] \qquad (12)$$

Where $[C]$ is referred to as the monodromy matrix or growth matrix associated with $[\phi(\tau)]$ and is a key matrix in determining the solution of interest.

A semi-analytical method is adopted to obtain the characteristic values of the matrices. Setting $\tau=0$ and let $[\phi(0)]=[I]$, from Eq. (12)

$$[C]=[\phi(T)] \qquad (13)$$

Thus the monodromy matrix is obtained, and the characteristic values can be obtained by solving the eigenvalue problem associated with that matrix.

4. RESULT AND CONCLUSION

Dynamic instability region is examined due to the nonstationarity of effective system stiffness comes from time-dependent non-conservative load applied in discrete manner. It turns out that there exist two types of instability. One is

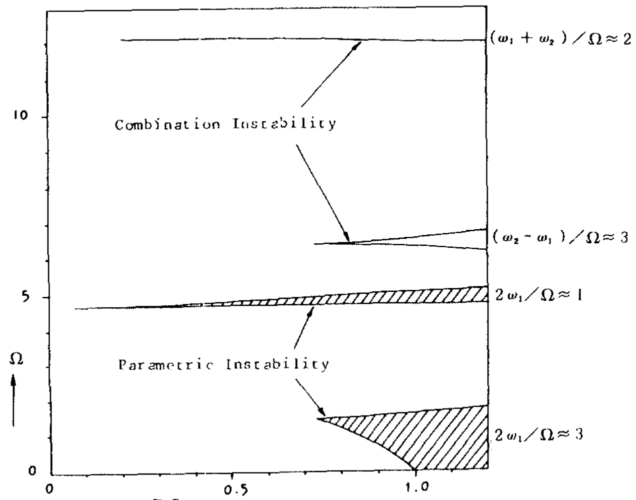

Fig. 3 Stability chart($\tau^*/T_P=0.4$, $C_i=0$, $\omega_1=2.467$ rad/sec, $\omega_2=22.207$ rad/sec)

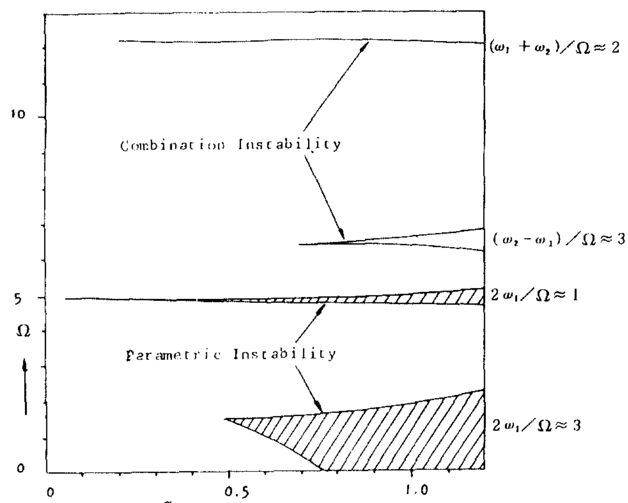

Fig. 4 Stability chart($\tau^*/T_P=0.5$, $C_i=0$)

parametric instability which occurs when the parametric resonance condition($n\omega_i/\Omega\approx k$, n and k are integer number, ω_i is the i-th normal mode frequency and Ω is parametric excitation frequency) is met. The other one is combination instability which occurs when the combination resonance condition [$(\omega_i\pm\omega_j)/\Omega\approx k$, ω_j is the j-th normal mode frequency] is satisfied. All of those regions are plotted in the stability charts, Fig. 3 and Fig. 4, when τ^*/T_P is 0.4, 0.5 respectively.

In order to confirm the result, numerical simulations are done for the stable and unstable regions. Fig. 5 shows that the system in the stable region ($\tau^*/T_P=0.5$, $\alpha_0=0.9$, $\Omega=4.847$) has a bounded response. On the other hand, as in Fig. 6, 7,8,9, the system in the unstable regions ($\tau^*/T_P=0.5$, $\alpha_0=0.9$, $\Omega=4.934$ and $\Omega=6.58$) show unbounded flutter type responses. The main conclusions obtained from this study are summarized below.

(1) When a non-conservative load is applied in discrete manner, the duration time of the non-conservative load is one of the main factors which affects the system's stability.

Generally, it can be said that the system is stable when the duration of applied load is quite remote from the half of the period.

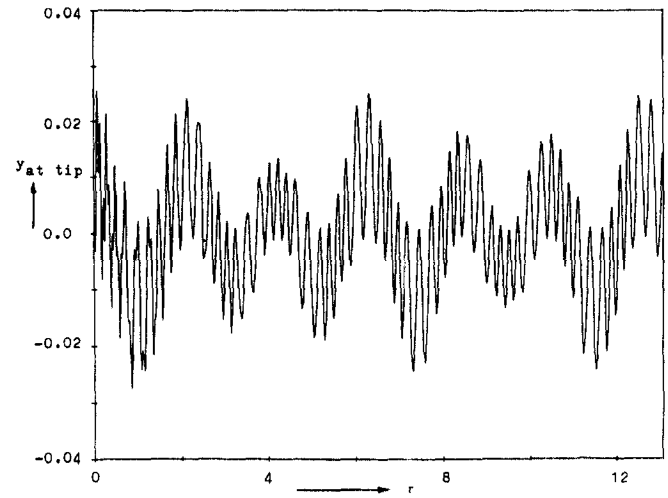

Fig. 5 Bounded response($\tau^*/T_P=0.5$, $\alpha_0=0.9$, $\Omega=4.847$, $C_i=0$)

(2) The higher the frequency of parametric excitation, the narrower the instability region becomes. The larger the amount of non-conservative load, the wider the instability region becomes.

(3) The first mode has a bounded response, for the parametric instability case ($2\omega_i/\Omega\approx 1$), the contribution of the third mode in response is negligible, while the second mode has flutter-type response.

In other words, parametric excitation energy is transferred to second mode, and this second mode contributes mainly to make system unstable.

(4) For combination instability ($(\omega_2-\omega_1)/\Omega\approx 3$), similar to the parametric instability, the second mode has a bounded response. Again, the third mode is vanishing and the first mode has unbounded response. This time, excitation energy is transferred to the first mode and the mode is responsible mainly for the system instability.

(5) Unlikely to the linear vibration, one of the unique phenomena which occurs in parametric vibration is that external energy pumped into system parametrically is directly transferred to one of the modes when certain condition is met. (For example, when the ratio between modal frequencies or combination of those and parametric excitation

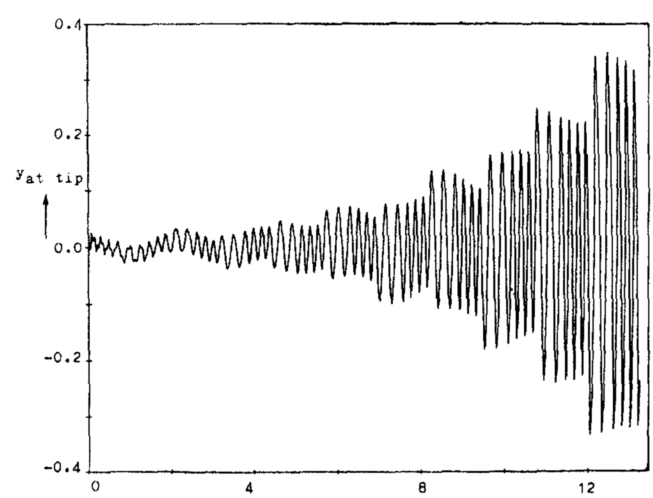

Fig. 6 Unstable response due to parametric resonance($\tau^*/T_P=0.5$, $\alpha_0=0.9$, $2\omega_1/\Omega\approx 1$, $C_i=0$)

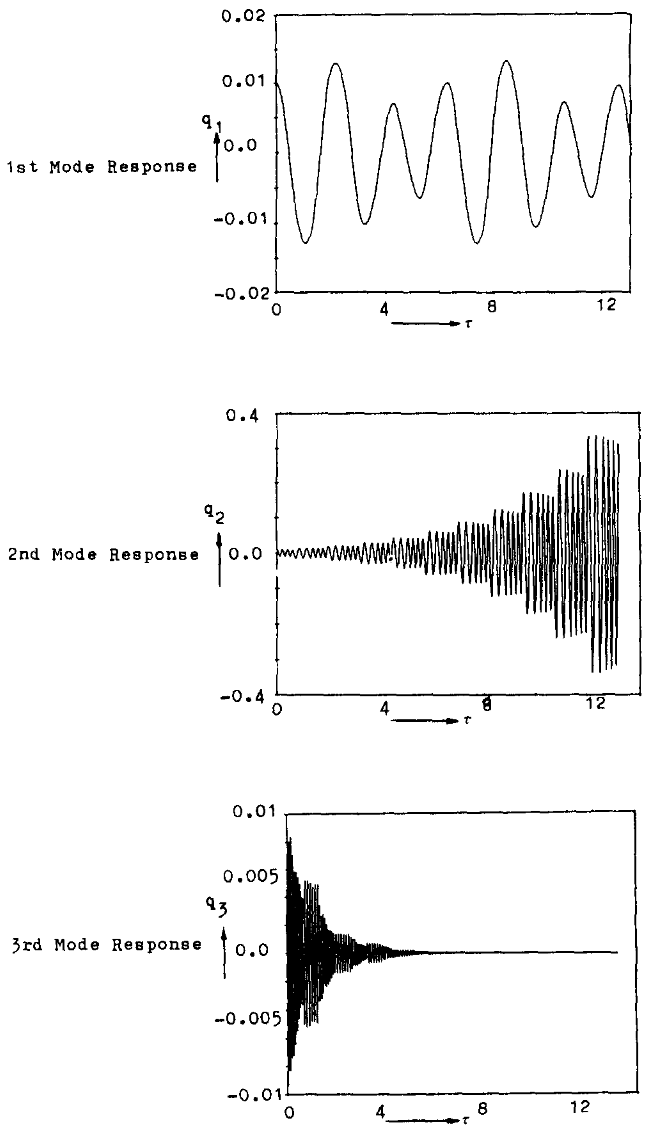

Fig. 7 Unstable modal response($\tau^*/T_P=0.5$, $\alpha_0=0.9$, $2\omega_1/\Omega\approx1$, $C_i=0$)

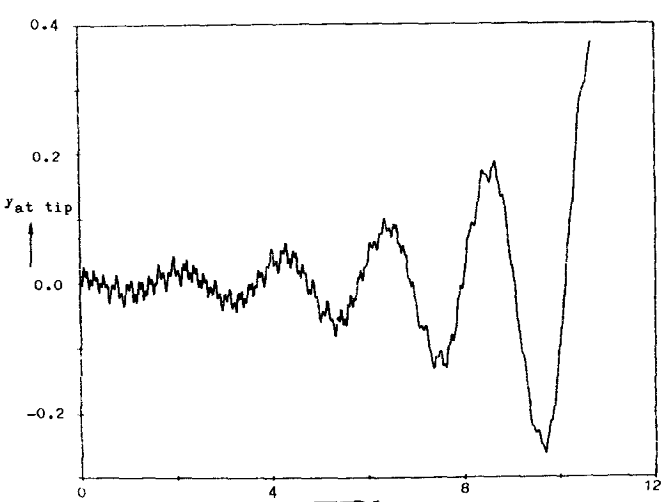

Fig. 8 Unstable response due to combination resonance($\tau^*/T_P=0.5$, $\alpha_0=0.9$, $(\omega_2-\omega_1)/\Omega\approx3$, $C_i=0$)

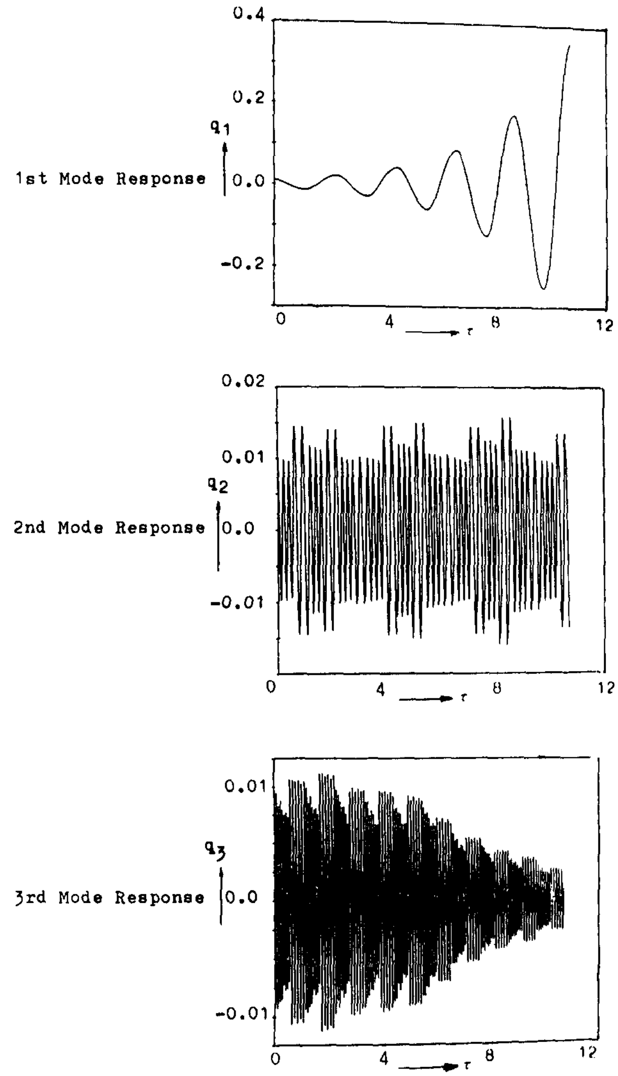

Fig. 9 Unstable modal response($\tau^*/T_P=0.5$, $\alpha_0=0.9$, $(\omega_2-\omega_1)/\Omega\approx3$, $C_i=0$)

frequency becomes an integer value)

Then those energy transferred modes become the source of nonlinearity in response.

REFERENCES

Bolotin, V. V., 1963, "Nonconservative Problems of the Theory of Elastic Stability", Pergamon Press.

Bolotin, V. V., 1964, "The Dynamic Stability of Elastic Systems", San Fransisco : Holden-Day, Inc.

Cesari, L., 1971, "Asymptotic Behavior and Stability Problems in ordinary Differential Equations", Springer-Verlag.

Chen, S. S., 1971, "Dynamic Stability of a Tube Conveying Fluid", Journal of the Engineering Mechanics Division, Proceedings of the ASCE, Vol. 97, pp. 1469~1485.

Coddington, E. A. and Levinson, N., 1955, "Theory of Ordinary Differential Equations", McGraw-Hill, New York.

Ginsberg, J. H., 1973, "The Dynamic Stability of a Pipe Conveying a Pulsatile Flow", International Journal of Engineering Science, Vol. 11, pp. 1013~1024.

Ibrahim, R. A., 1978a, "Parametric Virbration, Part III, Current Problems(1)", The Shock and Vibration Digest, Vol.

10, No. 3.

Ibrahim, R. A., 1978b, "Parametric Vibration, Part IV, Current Problems(2)", The Shock and Vibration Digest, Vol. 10, No. 4.

Leipholz, H., 1970, "Stability Theory", Academic Press.

Leipholz, H., 1975, "Die Direkte Methode der Variationsrechnung und Eigenwertprobleme der Technik", G. Braun Karlsruche.

Leipholz, H., 1980, "Stability of Elastic System", Sijthoff & Noordhoff.

McLachlan, N. W., 1964, "Theory and Application of Mathieu Functions", New York : Dover Publications.

Ro, Kwang Choon and Park, Young Pil, 1986, "A Study on the Stability of the Cantilever Beam with Several Masses Subjected to a Non-Conservative Force", Transactions of KSME, Vol. 10, No. 1, pp. 43~49.

Tso, W. K. and Fung, D. P. K., 1971, "Dynamic Instability under the Combined Actions of Non-Conservative Loading and Base Motion", Journal of Applied Mechnics, Trans. of ASME, Dec. pp. 1074~1076.

Ziegler, H., 1968, "Principles of Structural Stability", Blaisdell Publishing Co.